Simple & Optimal Greedy Online Contention Resolution Schemes

Vasilis Livanos

Department of Computer Science, University of Illinois at Urbana-Champaign

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Model

▶ **Input:**

Linear or Concave Program:

$$\max\ c^T z \quad \text{s.t.}\ z \in \mathcal{P}(\mathcal{C}),\ 0 \le z_i \le 1 \qquad \text{or} \qquad \max\ g(z) \quad \text{s.t.}\ z \in \mathcal{P}(\mathcal{C}),\ 0 \le z_i \le 1$$

Point $x \in$ polytope $\mathcal{P}(\mathcal{C})$.

▶ **Goal:**

Round x while respecting constraint $\mathcal{C}$.

▶ **Idea:**

Draw random $R \sim \text{Prod}(x)$ *(active elements)*, i.e. $i \in R$ *independently* with probability x_i $\forall i$.

R might be infeasible.

▶ **Idea # 2:**

"Drop" elements to obtain feasible $\pi(R) \subseteq R$.

▶ **Output:**

Algorithm $\pi : 2^{\mathcal{N}} \to 2^{\mathcal{N}}$ s.t.

$$\Pr[i \in \pi(R) \mid i \in R] \ge \alpha$$

for largest α possible $\implies$ α-approximation.

▶ **Challenges:**

Decide whether $i \in R$ or not **online**.

Arrival order of i's is determined by **almighty adversary**.

Cannot change past decisions.

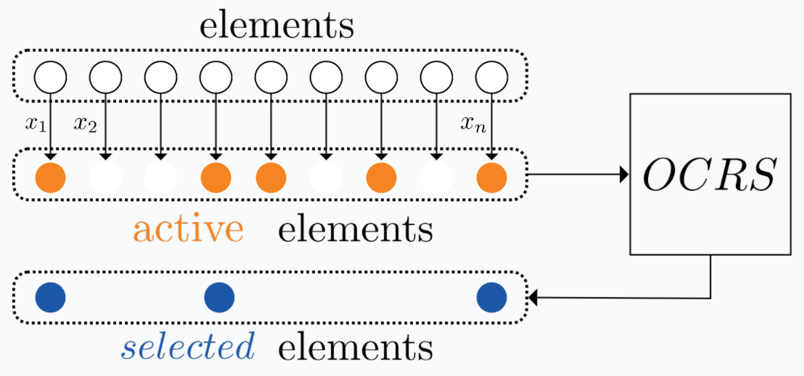
Background

Algorithm π is called an **Online Contention Resolution Scheme (OCRS)**. To compare against almighty adversary, we need:

Definition 1.

A **Greedy OCRS** π:

- ▶ Defines a subfamily of feasible sets $\mathcal{F}_{\pi,x} \subseteq \mathcal{C}$.
- ▶ When $i \in R$ arrives, π selects i if $\pi(R_{i-1}) + i \in \mathcal{F}_{\pi,x}$.



$\min_i \Pr[i \in \pi(R) \mid i \in R]$ is **selectability** of π

Previous best: $1/4$-selectable Greedy OCRS for matroids [FSZ '16].

Theorem 1

There exists a $1/e$-selectable greedy OCRS for

- ▶ rank-1 matroids.
- ▶ partition matroids.
- ▶ transversal matroids.

Theorem 2

No greedy OCRS can be $\left(1/e + \varepsilon\right)$-selectable, even for rank-1 matroids.

Algorithm

▶ **Main Idea:**

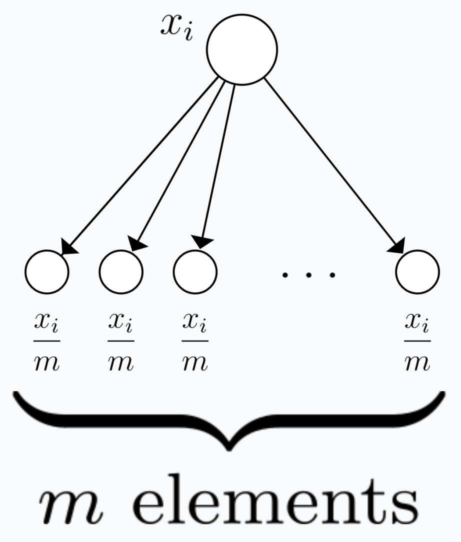
Create set $T \subseteq \mathcal{N}$, where $i \in T$ independently with probability $\frac{1-e^{-x_i}}{x_i}$.

I.e. $\mathcal{F}_{\pi,x} = \{\{i\} \mid i \in T\}$.

When i comes, *greedily* accept it if $i \in R \cap T$.

▶ **Intuition:**

Simulates splitting i into m elements with probability x_i/m each, as $m \to \infty$.

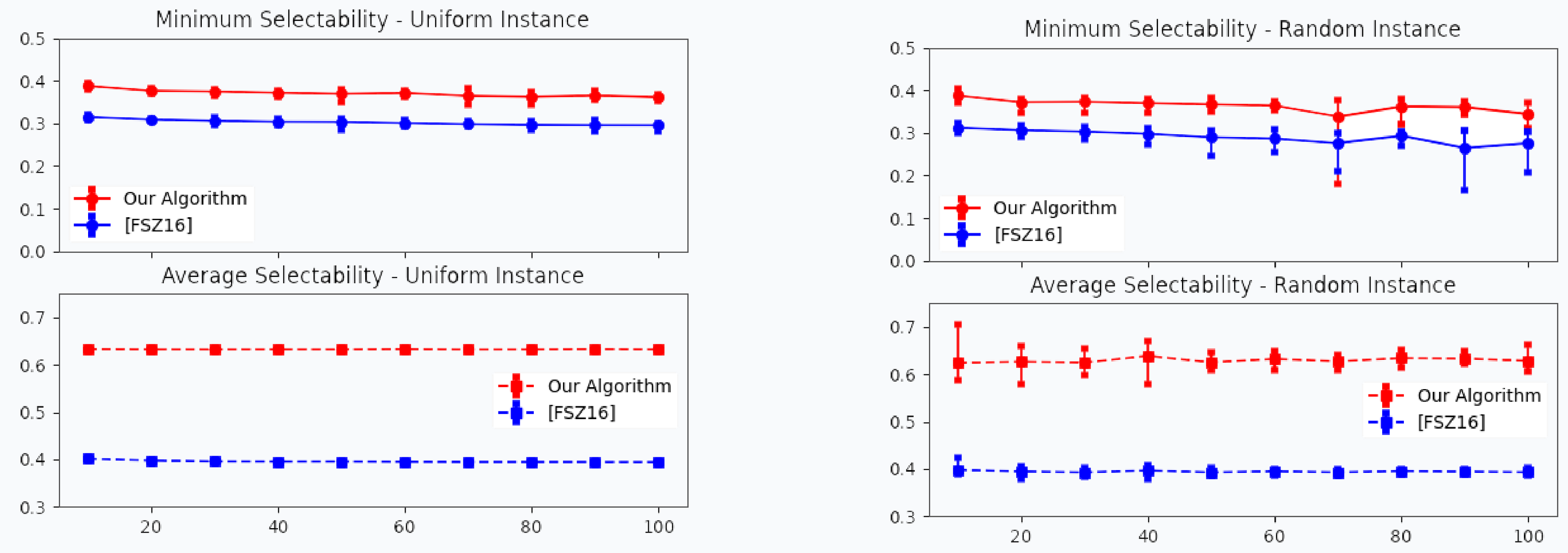


▶ **Generalization:**

Doesn't work for transversal matroids $\implies$ need different π.

Better $(1 - 1/e)$-approximation when $|N(u)| \ge 3$ for all elements u.

Experiments



N $= 5$ to 100, ITERATIONS $= 200,000$, REPETITIONS $= 10$